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A SIMPLIFIED DETERMINATION OF THE SHAPE OF A SUGAR BEET TAP-ROOT

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ABSTRACT

This work presents theoretical and empirical analysis of the usefulness of the selected geometrical models, for the purpose of determination of the sugar beet root shape, based on the available root parameters: diameter, length and the bulb root height. Analysis covered four models in the form of: a spherical sector, composed of a conic and a half of a sphere, composed of a conic and a spherical cap, composed of a conic and a half of an ellipsoid. Within the scope of the assumption of this paper it was proved, that the shape of a tap-root can approximate the simplified model comprising a conic and a half of an ellipsoid.

Introduction

Sugar beet roots are a valuable raw material used for production of sugar and lately for generation of renewable energy and the object of numerous experimental researches. In numerous experiments, dimensions of a root, indispensable for environmental assessment of a morphological structure are measured very rarely (Byszewski, 1979; Guerif et al., 1986; Gutmański, 1991; Ostrowska et al., 2002; Ostrowska and Artyszak, 2005; Przybył, 1998). The literature informs that the soil properties and yield of sugar beet roots is varied on account of soil and climatic conditions and the cultivation systems with a variable number of treatments and crossings of tractor wheels and agricultural machines (Hanse et al., 2011; Koch et al., 2008; Krauze et al., 2009; Sztukowski and Błaszkiewicz, 2002; Tomanowa et al., 2006). The above factors may lead to differentiation of the sugar beet root system and important technological morphological properties of roots (Błaszkiewicz et al., 2003; Dieckmann et al., 2006, Hoffmann and Jungk, 1996; Gajewnik 2013; Koch et al., 2008; Krauze et al., 2009; Lipiec et al., 2012; Ostrowska et al., 2002; Ostrowska and Artyszak, 2005; Schlinker et al., 2007; Scott et al., 1973). It is confirmed by numerous observations which indicated various shapes and sizes of sugar beet in agricultural practice. The advanced computer techniques applied more frequently in order to support the production management require formalized models of agricultural objects including models of the shape of sugar beet roots. Such models, supported by the shape of root parameters may be crucial in agricultural engineering and may serve for approximated and theoretical estimations of the size of root mass losses which take place during machine harvesting as a result of improper topping or breaking roots or for determination of the cubic capacity of storing and transportation means (Gutmański, 1991; Karwowski, 1982; Przybył, 1998).

Scientific centres keep results of researches carried out during harvesting of sugar beets in the form of basic dimensions of the tap - root: the biggest diameter, the total length of a root and the bulb root height. Thus, a question arises whether one may on this basis carry out determination of the simplified model of the sugar beet roots shape with the use of simple solids and geometrical formulas. Only general definitions of the tap-root shape, which may be found in the subject literature, given in the aspect of technological requirements, e.g. pointed-conical (Byszewski, 1979) or wedge-shaped (Ostrowska and Artyszak, 2005), have more orientative meaning than the scientific and practical ones. Thus, even approximated and general theoretical description of the shape of the sugar beet tap-root seems to be necessary. As a result of the above discussions, the objective of the paper was formulated.

The objective of the paper

The objective of the paper is to determine models of the sugar beet tap-roots shape based on a diameter and the total length of a root and the bulb root height as well as their assessment made based on a comparison of the determined parameters of the model and real solids.

Material and methodology of research

Determination of the sugar beet shape models is based on the idea of simple geometrical solids including available parameters of the morphological structure of roots, such as: length of the bottom part H of a root, height of its bulb root h and the highest diameter D_m (fig.1) obtained from field measurements. It was assumed to consider a root as consisting of two parts: lower conical and upper called a bulb root. Parameters describing these parts of the morphological structure of the tap-root were defined as follows. A cross section of a root in the widest place was assumed in the form of a circle, a diameter of which D_m is made of the average value from at least five measurements. A root part below its highest diameter was called a lower part – conical, length of which H is made of a fragment from the place where conical parts of a root meet to its highest diameter – the place where a root meets a measuring device. A bulb root, the height of which is made of a fragment from its highest part to the diagonal cross section in the place of the highest diameter, constitutes the upper part of a root.

The shape of the sugar beet tap root was described with one or two basic spatial geometrical solids. It was assumed that based on two available parameters D_m and H and for its pointed-conical shape (Byszewski, 1979) or the wedge shape (Ostrowska and Artyszak, 2005) required on account of technological aspects, a conic should reflect the lower part of the sugar beet root. As a root head model, a spherical cap, a half of a sphere and a half of ellipsoid were considered. As a result of the above, four variants of the geometrical model of the tap root of a sugar beet (fig. 1-3) were suggested in the following form:

I – a spherical sector,

II – a cone and a half of a sphere,

III – a cone and a spherical cap,
IV – and a cone as a half of an ellipsoid.

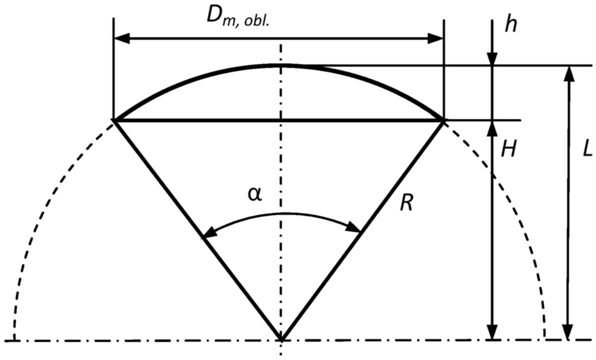

Figure 1. Parameters of sugar beet root model as a spherical sector

Mathematical formulas for determination of the root model volume, for the assumed geometrical solids were presented in table 1. Model I of the root shape in the form of a spherical sector and the describing parameters (where $R = L$) were presented in figure 1. Model II of the sugar beet root shape made of the half of a sphere, which adheres to the base of the reverse conic and the describing parameters were presented in figure 2. In this model, the sugar beet bulb root shape was described with a half of a sphere for the case, when the head height is similar to the half of a diameter of its base ($h \approx ½\ D_m$).

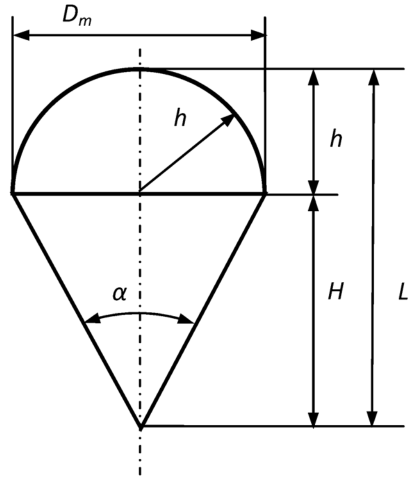

Figure 2. Parameters of sugar beet root model, composed of conic and half of sphere

If two above cases do not take place, the sugar beet shape may be described with a reversed conic and a spherical cap, which adheres to it (model III) or with a reversed conic and a half of ellipsoid which adheres to its base (model IV) (fig. 3).

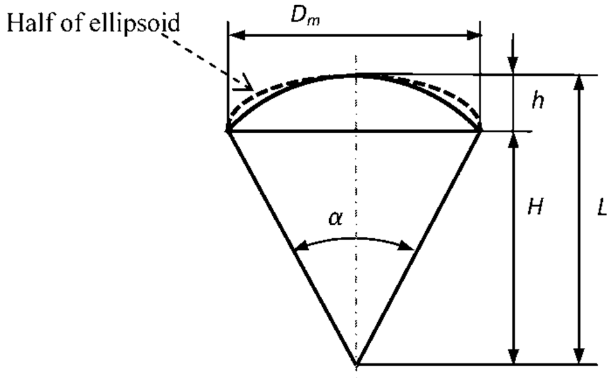

Figure 3. Parameters of sugar beet root model, composed of conic and spherical cap or composed of conic and half of ellipsoid

Evaluation of the usefulness of the suggested models for description of the sugar beet tap root shape was made in the theoretical and empirical analysis of relative errors of approximation determined based on the dimensions of roots obtained from calculations and field research and determined volumes of the model solids of roots.

Table 1

Geometrical formulas for the volume of analyzed geometric solids that form the model of the tap-root of sugar beet

Destination of the tap-root model	Geometric model of the tap-root	Formula for the volume of the conic part of the tap-root	Formula for the volume of the bulb root	Formula for the volume of the tap-root
I	Spherical sector	–	–	$V_c = \frac{2}{3}\pi h L^2$
II	Conic and half of sphere		$V_g = \frac{2\pi}{3} h^3$	$V_c = \frac{\pi}{3}\left(\frac{D_m^2}{4} H + 2h^3\right)$
III	Conic and spherical cap	$V_s = \frac{\pi}{12} D_m^2 H$	$V_g = \frac{\pi}{6} h \left(\frac{3}{4} D_m^2 + h^2\right)$	$V_c = \frac{\pi}{6}\left[\frac{D_m^2}{2} H + h\left(\frac{3}{4} D_m^2 + h^2\right)\right]$
IV	Conic and half of ellipsoid		$V_g = \frac{\pi}{6} D_m^2 h$	$V_c = \frac{\pi}{6} D_m^2 \left(\frac{H}{2} + h\right)$

As a measure of accuracy, a relative error e_r (w %) of compliance of the root head radius $\frac{D_m}{2}$ and its height h, calculated from formula1:

$$e_r = \left(\frac{D_m}{2} - h\right) 2D_m^{-1} \cdot 100 \quad (1)$$

and a relative error e_D (w %) of compliance of the bulb root diameter obtained from calculations $D_{m,obl.}$ and diameter obtained from empirical tests D_m, calculated from formula 2:

$$e_D = \left(\frac{D_{m,obl.} - D_m}{2}\right) 2D_{m,obl.}^{-1} \cdot 100 \quad (2)$$

A diameter of the bulb root $D_{m,obl.}$, required for calculation 2, was calculated from formula 3 for the spherical sector:

$$D_{m,obl.} = 2\sqrt{(2L - h)h} \quad (3)$$

Parameters h, L and D_m indispensable for validation researches were determined in the research of a 3-year field experiment carried out in light soils – LS loamy sand (acc. to the FAO classification), with a method of random blocks, in four repeats of blocks (I-IV), for traditional technology of sugar beet – Sonia variety cultivation. Sugar beets were cultivated in rows with 0.45 m spacing and distance in the row of 0.18 m. The analysed parameters of roots were measured during harvesting after the removal of leaves with a special measuring device. Results presented in table 2 constitute average values from measurements of 10 roots which were selected at random, made in each block.

Table 2
Results of calculation and empirical investigation of tap-root parameters of sugar beet

Year of investigation	Repetition of empirical research block	Root dimensions (cm) (m)				Relative error e_r, (%)	$\frac{D_{m,obl}}{2}$ [cm]	Relative error e_D, (%)
		D_m	h	H	L			
1	I	10.70	3.33	17.08	20.41	37.8	11.1	70.2
	II	10.40	2.91	17.29	20.20	44.0	10.4	72.2
	III	10.16	2.91	16.58	19.49	43.2	10.2	71.6
	IV	10.50	3.45	17.75	21.20	34.3	11.6	70.2
2	I	11.06	3.91	19.91	23.82	29.3	13.1	70
	II	10.60	3.62	19.49	23.11	31.7	12.4	70.9
	III	11.40	3.88	19.49	23.37	31.9	12.9	69.9
	IV	10.8	3.24	17.95	21.19	40.0	11.3	71.2
3	I	8.16	4.07	24.91	28.98	1.0	14.8	72.5
	II	8.26	4.07	21.79	25.86	1.4	13.9	70.1
	III	8.86	4.13	21.54	25.67	1.5	14.0	70.2
	IV	10.04	3.96	23.84	27.80	21.1	14.3	72.3

Table 2 includes determined values of relative errors e_r and e_D. Whereas, a volume of geometric solids for the analysed models of a root, calculated from formulas presented in table 1, were presented in table 3.

Results of research and their analysis

A lower part of a root was described based on only two parameters D_m and H with the use of a conic, which is compliant to the requirement set forth in literature, that the shape of this part of a root should be conic (an oval shape is assigned to a fodder beet) (Byszewski, 1979; Ostrowska and Artyszak, 2005).

Table 3
Volumes of sugar beet tap-root (in cm^3), calculated from the analyzed geometric formulas

Year of investigation	Research block	Volume of conic shape of tap-root	Model			
			I	II	III	IV
		V_s	V_c	V_c/V_g	V_c/V_g	V_c/V_g
1	I	511.8	2903.8	832.3/320.5	741.5/229.7	711.3/199.5
	II	489.3	2485.6	783.6/294.3	625.7/136.4	654.0/164.7
	III	447.8	2313.9	722.2/274.4	578.6/130.8	605.0/157.2
	IV	512.1	3245.8	815.0/302.9	674.3/162.2	711.2/199.0
2	I	637.3	4644.1	991.3/354.0	844.3/207.0	887.6/250.3
	II	573.0	4047.1	884.6/311.6	757.4/184.4	802.9/229.9
	III	662.8	4435.9	1050.5/387.7	891.3/228.5	926.7/263.9
	IV	547.8	3045.4	877.4/329.6	730.1/182.3	745.6/197.7
3	I	423.4	7155.3	560.4/137.0	562.5/139.1	561.8/138.4
	II	389.0	5697.6	536.5/147.5	532.0/143.0	534.3/145.3
	III	442.4	5696.9	624.4/182.0	606.5/164.1	612.1/169.7
	IV	628.8	6406.5	893.6/264.8	847.6/218.8	837.7/208.9

V_c – volume of tap root
V_g – volume of bulb root

Selected models for imaging the bulb root in the form of a spherical cap, half of a sphere and a half of an ellipsoid. Assessment of precision of models was made for subsequent years of research, which characterize with a considerable variability of the root dimensions. In order to extend the assessment of the usefulness of models, the analysis included diversity of dimensions of roots which occurs in repeats of the field research in blocks. Results of the three-year research included in table 2 show that the ratio of the bulb root height h to its radius of the base is varied, which justifies why various geometric solids for description of the bulb shape were accepted for analysis. In the 3rd year of research (table 2: block I, II and III) the radius and height of the root head are very similar (a determined relative error is lower than 1.5%), whereas in the 1st and 2 nd year of research differences in these parameters are even more than 30%. Moreover, the data presented in table 2 show that in each year of three-year research, the beet head height was lower or at least equal to its base radius, which justifies acceptance of the suggested halves of geometric solids for imaging the root head shape.

In the assessment of model I, it was assumed that the compliance of the empirically determined sugar beet diameter D_m and diameter $D_{m,obl.}$ (table 2) calculated based on parameters L and h of a root (where $L=R;\ R=H+h$) from the formula 3 may decide on the usefulness of the suggested geometric solid in the form of a spherical sector for imaging of the tap-root. A relative error calculated from formula 2 was the basis of analysis. The data presented in this table (col. 3, 8 and 9) prove that the spherical sector very weakly describes the sugar beet root. Differences in the calculated and empirically determined diameter of a beetroot are highly non-compliant, burdened with a relative error which is more than 70%. Moreover, the calculated volume of a root (table 3) decisively diverges from the one calculated for other models. These analyses show clearly that the spherical sector is not useful in imaging the tap-root shape.

In the assessment of model II of the root, verification was carried out which aimed at determination whether a half of a sphere may describe the bulb root, i.e. whether the sugar beet bulb height is similar to the radius of its base $h \approx ½ D_m$. The analysis of the calculated relative error (table 2, column 7) shows that only in the recent year of research, a quite high compliance of two empirically determined parameters occurs h and $½ D_m$ (table 2, col. 3 and 4), thus a description of the bulb root with a sphere may be accepted.

In the case of considerable differences of the head height and its radius (table 2: year 1 and 2 of the research, col. 3 and 4) for description of the bulb root, the usefulness of the spherical cap or a half of ellipsoid was considered (table 1: model III and IV). The data included in table 3 (col. 6 and 7) show that differences in the volume of the bulb root calculated with formulas for a spherical cap and a half of an ellipsoid are higher if the bulb root height and a diameter are different, i.e. when the bulb root is more flattened, which can be observed particularly in the 1st and 2nd year of research (table 2, col. 3 and 4). It should be assumed that higher reported values of the calculated half of an ellipsoid are more justified because they result from better approximation of the ellipsoid surface to the bulb root in the place of its highest diameter. Since, the beetroot surface is more rounded in this place than the angular, imaged spherical cap (fig. 3). Description of the root head with a half of an ellipsoid may be improved even by 20% with reference to a spherical cap (tab. 2), which results from the differences of the calculated values of volume for both figures. It may be assumed, within assumptions of this paper, that the suggested simplified model of a root made of a conic and a half of an ellipsoid (model IV) from among the analysed ones, provides the best image of the sugar beet root shape. Model IV replaces also model II when the values D_m and h get close and the shape of a half of an ellipsoid changes into a half of a sphere.

Obtaining models of the sugar beet roots with higher precision makes additional scientific work necessary based on more detailed experimental measurement of the shape and volume. However, for realization of many practical, teaching and general engineering objectives, lower precision of assessment obtained with considerably lower costs of research and analysis may be sufficient.

Conclusion

The paper analysed the sugar beet roots models in the form of one geometrical solid – a spherical sector or comprising two geometrical solids, where a bottom part of the root is imaged by a reversed conic and the head is alternative presented in the form of: a spherical cap, a half of a sphere or a half of ellipsoid. It was assumed that the pointed-conical or wedged shape of the bottom part of the sugar beet root, required on account of technological reasons, should describe a conic. Accepting halves of the suggested geometrical solids for imaging the rood head shape justify results of the root dimensions from a three-year research, which show that the beet head height is lower or at least equal to its radius of the base. The comparative analysis of parameters of the geometrical figures and their values, which were empirically determined, showed within the assumptions of the paper that the spherical sector does not sufficiently describe the shape of the tap-root of sugar beet. The root shape may be described with the use of a model comprising a conic and a half of an ellipsoid which adheres to its base.

For issues which require higher precision of the sugar beet models, further research and analyses carried out based on detailed experimental measurements of the shape parameters and root volume are recommended.

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UPROSZCZONE WYZNACZANIE KSZTAŁTU KORZENIA PALOWEGO BURAKA CUKROWEGO

Streszczenie. W pracy przedstawiono rezultaty teoretyczno-empirycznej analizy przydatności wybranych modeli geometrycznych do opisu kształtu korzenia palowego buraka cukrowego, wyznaczanych w oparciu o dostępne wymiary korzenia: średnicę i jego długość oraz wysokość główki. Analizowano cztery modele w postaci: wycinka kuli, złożonego ze stożka i połowy kuli, stożka i czaszy kulistej oraz ze stożka i połowy elipsoidy. Wykazano, w zakresie założeń pracy, że stożkowy korzeń palowy może być opisany za pomocą uproszczonego modelu złożonego ze stożka i opartej na jego podstawie połowy elipsoidy.

Słowa kluczowe: burak cukrowy, korzeń palowy, kształt, model geometryczny