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METHOD OF MATHEMATICAL MODELLING OF THE SURFACE OF THE EGG SHELL SHAPE, EGG YOLK AND AIR CHAMBER OF CHICKEN EGGS

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ABSTRACT

The paper presents a method of mathematical modelling of the shape of the surface of the egg shell, egg yolks and air chamber. For modelling of the shape, eggs with dimensions: length 60; 57.2; 56.4 mm, width 47.1; 42.3; 41 mm and thickness of 46; 41.1 and 39.2 mm were selected. Two Bézier patches were used to map the shape of the surface of the egg shell, yolk and the air chamber. Calculation and visualization was carried out in Mathcad computer program. The developed mathematical model of the 3D shape of chicken eggs and its basic elements can be used for modelling the processes occurring in their production and processing.

Introduction and objective of the paper

Production of chicken eggs in Poland within 2006-2013 was at the level of 10,393 million pieces (MRSP, 2013). Siepka et al. (2010) emphasize that the scope of use is wide. They are widely used in the food, pharmaceutical, cosmetics, chemical and feed industry. The research carried out by Śmiechowska and Pogórniak (2013) shows that eggs from a laying hen of greenleg partridge hen breed have length from 50 to 61 mm (average 54.7 ±2.52 mm), width from 38 to 44 mm (average 41 ±1.52 mm) and the shape index 1.24-1.40 (average 1.31 ±0.036). According to Shultz (1953) heredity of the shape index of chicken eggs is between 0.11 and 0.19. Mass of a chicken egg is between 40 to 80 g. Eggs with mass within 58 to 60 g are the most popular. Average thickness of a shell is 0.3 mm. A percentage share of egg white is 55.5%, of yolk 31.9% and a shell 12.3%. Height of the air chamber in fresh egss is within 4 to 6 mm. The egg shape is a feature that identifies its origin (Preston, 1968). According to Bardyn and Krysiak (2013), a cross-section of a chicken egg is elliptically shaped and narrowed at one end. A yolk is rounded and is centrally located. Calik (2013) determined that storing conditions of eggs affect both decrease of the egg mass, yolk dimensions and air chamber as well as the egg white content.

Research on the quality of eggs on account of freshness, improper shape, mechanical damages, blood stains inside an egg are carried out automatically with technologies related to computer graphics with the use of neuron networks. The obtained research results are used for creation of databases of eggs with correct and incorrect structure (Arivazhagen et al., 2013). In production and food processing of eggs, collection, washing, sorting and packing are basic technological operations, for which the egg shape is of basic importance. The mentioned technological operations are manually carried out in small farms and in innovative large scale production they are automatised and roboted (Garcia-Alegre et al., 1997; 1998; 2000; Patel et al., 1998). In the processes of sorting, detection of egg defects and their irregular shapes non-destructive technologies with the use of vision systems and databases of shapes and eggs image are applied (Garcia-Alegre et al., 1998). In the robots used for packing eggs, vacuum pneumatic suckers, the shape of which is selected to the egg shape, are used. Structure of sorting machines, elevator belts, dosing wheels, lantern pinions which transfer eggs to rod cross and diagonal conveyors, counting systems, which assess the quality, incubators, baskets, containers etc., depends greatly on the egg shape. Modern, computer aided methods of designing machines and devices for a poultry force constructors to know geometric properties of processed eggs. Determination of the egg shape in a contractual manner does not suffice, e.g. elyptic, oval. Designers are provided with useful tools for description of the body shape by computer graphics (Kiciak, 2000; Foley et al., 2001). Mieszkalski (2011) used a parametric spatial curve and a four-rod network spread on the external surface of the modelled body for description of plant raw materials.

Keshavarzpour (2011), Rashidi and Gholami (2011), Rashidi and Keshavarzpour (2011) suggested models of linear regression, by means of which they expressed a relation between the egg mass and its geometrical parameters. Many works concern the description of the cross section outline in the two-dimensional system on the plane. Mónus and Barta (2005) and Barta and Székely (1997) suggest function $y=f(x)$, which is a thrid degree polynomial for description of the egg outline. Based on digital pictures of ostrich eggs Nedomová and Buchar (2013) with the use analysis of the image, using a technology for detecting edges, obtained points coordinates for egg profiles, which were approximated with Fourier's row. Nishiyama (2012) says that longitudinal cross sections of eggs are neither rounded not elyptic but oval, therefore for description of the shape of longitudinal cross section outline, he used Cassini's oval.

The issue which must be solved is development of a method, with which 3*D* description of the eggs shape and their components would be possible.

The objective of the paper is developing a method of modelling the chicken egg shape and its basic elements (shell, yolk, air chamber) with the use of Bezier patches.

Research material

Chicken eggs were the material for the research, which come form OLDAR from Sokołow from a cage breeding from 2014. Three different egg shapes were selected, marking them as I, II, III. Eggs were photographed with a digital camera Lumix Panasonic DMC – TZ3 in the JPEG format. Basic shell dimensions (long axis, short axis I, short axis II, shell thickness), yolks (diameter), air chamber (length, width, height) were measured with a caliper with precision up to 0.1 mm. The size of an egg and air chamber were determined after boiling. Results of measurements of the selected chicken eggs were presented in table 1.

Table1
Basic dimensions of the selected chicken eggs

Name of a component of an egg	Name of a dimension	Results of basic dimensions of components of eggs (mm)		
		I	II	III
Shell	Long axis	60	57.2	56.4
	Short axis I	47.1	42.3	41
	Short axis II	46	41.1	39.2
	Shell thickness	0.5	0.5	0.4
Yolk	Diameter	32.2	29.8	29.1
Air chamber	Length	8.2	7.4	7.1
	Width	8.1	7.2	7
	Height	3.4	3.1	2.6

Description of the method

Modelling the shape of the egg shell body was carried out with the use of Bezier patches. Parametric equations of coordinates of the Bézier patches in the matrix record take the following form (Kiciak, 2000; Foley et al., 2001):

$$x(s,t) = T^T \cdot M^T \cdot G_x \cdot M \cdot S \tag{1}$$

$$y(s,t) = T^T \cdot M^T \cdot G_y \cdot M \cdot S \tag{2}$$

$$z(s,t) = T^T \cdot M^T \cdot G_z \cdot M \cdot S \tag{3}$$

The parametric representation of the area *x=x(s, t), y=y(s, t), z=z(s, t)* depends on parameters *s* and *t* (vectors 4, 5). In equations (1, 2, 3) a base matrix of the Bézier patch occurs *M* (6).

$$T = \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix} \tag{4}$$

$$S = \begin{bmatrix} s^3 \\ s^2 \\ s \\ 1 \end{bmatrix} \tag{5}$$

$$M = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

Elements of three matrices of Bézier patch geometry G_x, G_y, G_z as matrices of geometric limitations are coordinates of 16 control points which are control points: The change of the coordinates of the control points decides on the patch shape and thus on the egg shape.

Matrices of geometry of Bézier patch *A* have the form:

$$GA_x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{bmatrix} \tag{7}$$

$$GA_y = \begin{bmatrix} 1{,}5 & 1{,}5 & 1{,}5 & 1{,}5 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 3 & 3 \\ 1.5 & 1.5 & 1.5 & 1.5 \end{bmatrix} \tag{8}$$

$$GA_z = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Whereas for the patch *B*: $GB_x = GA_x$, $GBy = GA_y$,

$$GB_z = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

The model of the external surface of the chicken egg shell consists of two connected Bézier patches *A, B*. Bézier patch *A* was used for modelling the shape of more rounded (smaller radius of rounding) part of an egg located in the end of its long axis, and patch *B* for modelling the shape of less rounded (bigger radius of rounding) of the egg part, located on the opposite side of patch *A*. Continuity and smoothness of the surface is obtained by connecting rod points of patches *A* and *B* at their edges (values of their coordinates are the same) and corresponding control points must be located on common straight lines. Determination of values of matrix elements of Bezier patch geometry G_x, G_y, G_z consists in adjusting the patch shape to the shape of egg piece.

Parametric equations, which determine coordinates of the network points of Bézier patch *A* have the following form:

$$xA_{i,j} = 6 \cdot t_j^2 - 4 \cdot t_j^3 \tag{11}$$

$$yA_{i,j} = -4{,}5 \cdot t_j + s_i^2 \left[27 \cdot t_j - 27 \cdot t_j^2\right] + 4{,}5 \cdot t_j^2 + s_i^3 \left[18 \cdot t_j^2 - 18 \cdot t_j\right] + 1{,}5 \tag{12}$$

$$zA_{i,j} = \left[36 \cdot t_j - 36 \cdot t_j^2\right] \cdot s_i + \left[36 \cdot t_j^2 - 36 \cdot t_j\right] \cdot s_i^2 \tag{13}$$

where:

$$s_i = i \cdot \frac{1}{N} \tag{14}$$

$$t_j = j \cdot \frac{1}{N} \tag{15}$$

$$i = 0 \ldots N \tag{16}$$

$$j = 0 \ldots N \tag{17}$$

$$t, s \in (0,\ 1) \tag{18}$$

N – matrix dimension (number of lines and columns).

Similarly to Bezier patch B the open forms of the parametric equations, which determine coordinates of the network points of the patch B are as follows:

$$xB_{i,j} = 6 \cdot t_j^2 - 4 \cdot t_j^3 \tag{19}$$

$$yB_{i,j} = -4{,}5 \cdot t_j + s_i^2 \left[27 \cdot t_j - 27 \cdot t_j^2\right] + 4{,}5 \cdot t_j^2 + s_i^3 \left[18 \cdot t_j^2 - 18 \cdot t_j\right] + 1{,}5 \tag{20}$$

$$zB_{i,j} = \left[-22{,}5 \cdot t_j^2 + 22{,}5 \cdot t_j\right] \cdot s_i^2 + \left[22{,}5 \cdot t_j^2 - 22{,}5 \cdot t_j\right] \cdot s_i \tag{21}$$

In order to obtain the body surface shaped similarly to an egg, a new matrix *XAB* should be formed by horizontal connection to a matrix *xA* of the matrix *xB* at the same number of lines in the added matrices. Similarly the matrices *yA* and *yB* are added, forming *YAB* and matrices *zA* and *zB*, obtaining matrix *ZAB*. Coordinates of points of surfaces obtained with the use of the matrix *XAB*, *YAB*, *ZAB* should be scaled in order to obtain surfaces similar with their dimensions to external and internal real surfaces of the egg shell. Scaling is carried out towards *X, Y, Z* axes of the coordinates system. The matrices, after scaling, which represent real shapes of the external surface of the chicken egg shell (*Xsz*, *Ysz*, *Zsz*), take the following form:

$$Xsz = \frac{XAB \cdot b}{\max(XAB) - \min(XAB)} \tag{22}$$

$$Ysz = \frac{YAB \cdot c}{\max(YAB) - \min(YAB)} \tag{23}$$

$$Zsz = \frac{ZAB \cdot a}{\max(ZAB) - \min(ZAB)} \tag{24}$$

The matrices, after scaling, representing real shapes of the internal surface of the chicken egg shell (*Xsw, Ysw, Zsw*), have the following form:

$$Xsw = \frac{XAB \cdot bw}{\max(XAB) - \min(XAB)} \tag{25}$$

$$Ysw = \frac{YAB \cdot cw}{\max(YAB) - \min(YAB)} \tag{26}$$

$$Zsw = \frac{ZAB \cdot aw}{\max(ZAB) - \min(ZAB)} - \tag{27}$$

The external dimensions (in mm) of a shell (*a* – length, *b* – width, *c* – thickness) and internal dimension of a shell (*aw* – length, *bw* – width, *cw* – thickness) of chicken eggs were placed in matrix 28.

$$\begin{bmatrix} a & aw \\ b & bw \\ c & cw \end{bmatrix} = \begin{bmatrix} 60 & 59 \\ 47{,}1 & 46{,}1 \\ 46 & 45 \end{bmatrix} \tag{28}$$

Model 3D of the chicken egg shell was presented in figure 1.

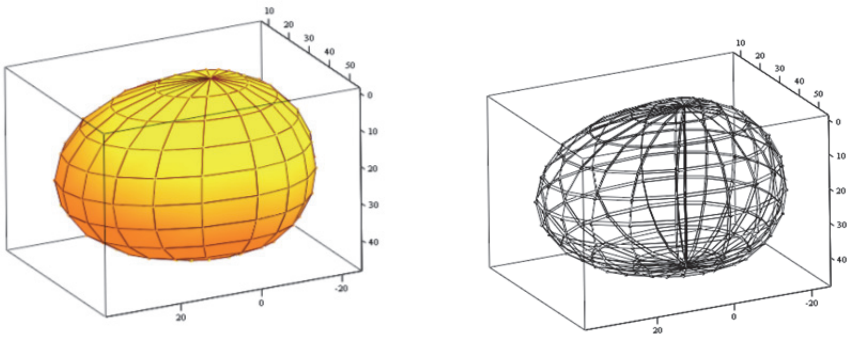

Figure 1. Surface and network model 3D of the chicken egg shell

In order to obtain the body surface shaped similarly to an egg yolk, a new matrix *XXż* should be formed by horizontal connection to the matrix *xB* of the matrix *xB* at the same number of lines in the added matrices. Similarly the matrices *yB* and *yB* are added, forming

YYż and the matrices *zB* and *-zB*, obtaining the matrix *ZZż*. The coordinates of points of surfaces obtained with the use of the matrix *XXż*, *YYż*, *ZZż* should be scaled in order to obtain surfaces similar with their dimensions to the external and internal real surfaces of the egg yolk. Matrices, after scaling, which represent real shapes of the external surface of the yolk (*Xż*, *Yż*, *Zż*) take the following form:

$$X\dot{z} = XX\dot{z} \cdot \frac{a\dot{z}}{\max(XX\dot{z}) - \min(XX\dot{z})} + \frac{b - a\dot{z}}{2} \tag{29}$$

$$Y\dot{z} = YY\dot{z} \cdot \frac{a\dot{z}}{\max(YY\dot{z}) - \min(YY\dot{z})} + \frac{b - a\dot{z}}{2} + (\max(YY\dot{z}) - \min(YY\dot{z})) \tag{30}$$

$$Z\dot{z} = ZZ\dot{z} \cdot \frac{a\dot{z}}{\max(ZZ\dot{z}) - \min(ZZ\dot{z})} \tag{31}$$

In order to obtain the surface of the body shaped similarly to the egg air chamber, a new matrix *XXp* should be formed by horizontal connection to the matrix *xA* of the matrix *xA* at the same number of lines in the added matrices. Similarly the matrices *yA* and *yA* are added, forming *YYp* and the matrices *zA* and *-Za*, obtaining the matrix *ZZp*. The coordinates of points of surfaces obtained with the use of the matrix *XXp*, *YYp*, *ZZp* should be scaled in order to obtain a surface similar with its dimensions to the external and internal real surfaces of the egg air chamber. The matrices, after scaling, which represent real shapes of the external surface of the air chamber (*Xp*, *Yp*, *Zp*) take the following form:

$$Xp = XXp \cdot \frac{ap}{\max(XXp) - \min(XXp)} + 0{,}5 \cdot (\max(X) - \min(X)) - \frac{ap}{2} \tag{32}$$

$$\begin{aligned} Yp = YYp \cdot \frac{bp}{\max(YYp) - \min(YYp)} + (\min(Y) + 0{,}5 \cdot (\max(Y) - \min(Y)) - \frac{bp}{2}) + \\ - (\max(YYp) - \min(YYp)) \end{aligned} \tag{33}$$

$$Zp = ZZp \cdot \frac{cp}{\max(ZZp) - \min(ZZp)} + \max(Z1) - \frac{cp}{2} \tag{34}$$

Dimensions (in mm) of the yolk diameter (*aż*) and the air chamber(*ap* – length, *bp* – width, *cp* – height) of the chicken egg placed in vector 35.

$$\begin{bmatrix} a\dot{z} \\ ap \\ bp \\ cp \end{bmatrix} = \begin{bmatrix} 32{,}2 \\ 8{,}2 \\ 8{,}1 \\ 3{,}4 \end{bmatrix} \tag{35}$$

Model 3*D* of the chicken egg body and air chamber was presented in figure 2.

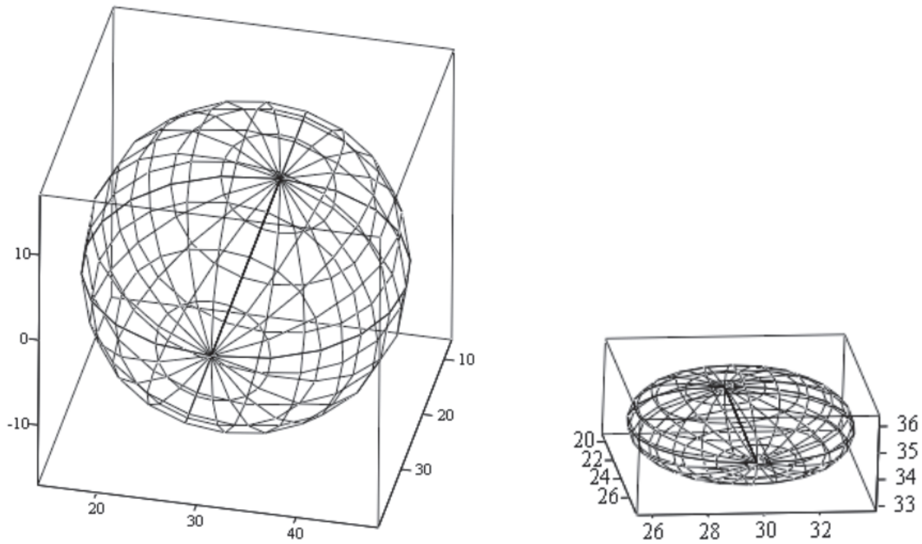

Figure 2. Models3D of the yolk body and air chamber of a chicken egg

*Model 3*D of a shell, yolk and air chamber of a chicken egg was presented in figure 3.

In order to verify a mathematical model, which describes the shape of chicken eggs I, II, III (dimensions in table 1) photographs of eggs and their models were taken folded on each other projected on the plane *YZ* and they were presented in the background of horizontal lines of a diagram (fig. 4). Horizontal lines cross the image of the model projection of the real body of an egg. Specific horizontal lines, crossing the outlines of projections, indicate the length of the indicated cross sections. The determined lengths of these cross sections for an egg and a model were compared and differences were described between them and a relative error was calculated (table 2).

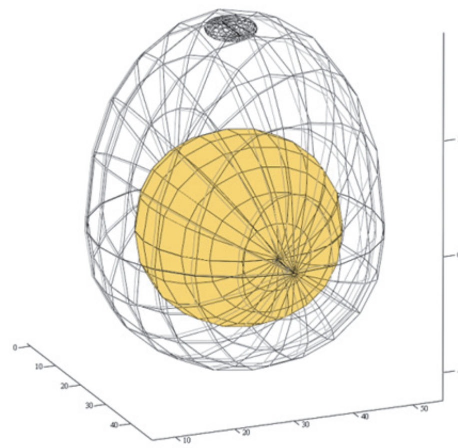

Figure 3. Model 3D of the body of a shell, yolk and air chamber of a chicken egg

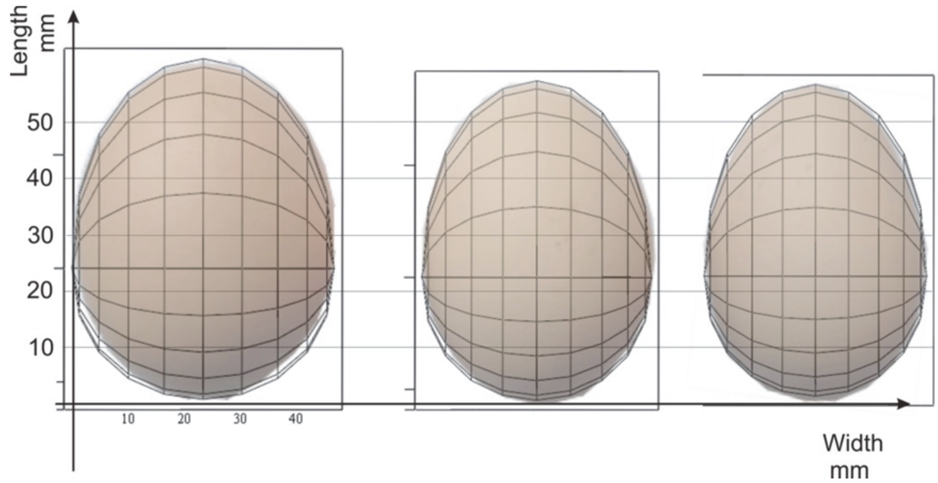

Figure 4. Projections on a plane YZ of models and real bodies of chicken eggs I, II, III

Comparison of the projections of eggs I, II, III and their models overlapping the plane YZ proves that themodels of chicken eggs reflect the eggs shapes chosen for modelling.

Table 2
Differences in the length measurement of models cross sections and lengths of marked cross sections of eggs and relative error

Distance between indicated cross sections of eggs (mm)	Differences between lengths of indicated cross sections of the model and length of the indicated cross sections of eggs (mm)			Relative error (%)		
	I	II	III	I	II	III
1	2	3	4	5	6	7
10	3.8	1.9	0.7	10.9	5.9	2.2
20	0.5	0	0	1.1	0	0
30	-0.4	-2.3	0	-0.9	-5.6	0
40	-1.3	-1.7	0.3	-3.1	-4.7	0.9
50	-0.9	-0.1	0.7	-2.7	-0.4	3.1

Table 3
Differences between long axis, short axis I, short axis IIof chicken eggs I, II, III and dimensions of their models

Dimension	Labelling of an egg	Differences between basic dimensions of eggs and dimensions of their models (mm)
Long axis	I	60 – [max(*ZszI*) – min(*ZszI*)]=0
	II	57.2 – [max(*ZszII*) – min(*ZszII*)]=0
	III	56.4 – [max(*ZszIII*) – min(*ZszIII*)]=0
Short axis I	I	47.1 – [max(*XszI*) – min(*XszI*)]=0
	II	42.3 – [max(*XszII*) – min(*XszII*)]=0
	III	41 – [max(*XszIII*) – min(*XszIII*)]=0
Short axis II	I	46 – [max(*YszI*) – min(*YszI*)]=0
	II	41.1 – [max(*YszII*) – min(*YszII*)]=0
	III	39.2 – [max(*YszIII*) – min(*YszIII*)]=0

Table 4
Differences between length, width, height of airing chamber and a diameter of egg yolk and the same dimensions of their models

Dimension	Differences between basic dimensions of air chamber, eggs yolk and dimensions of their models (mm)		
	I	II	III
	Air chamber		
Length	8.2 – [max(*XpI*) + – min(*XpI*)]= 0	7.4 – [max(*XpI*) + – min(*XpI*)]= 0	7.1 – [max(*XpI*) + – min(*XpI*)]= 0
Width	8.1 – [max(*YpI*) + – min(*YpI*)]= 0	7.2 – [max(*YpI*) + – min(*YpI*)]= 0	7 – [max(*YpI*) + – min(*YpI*)]= 0
Height	3.4 – [max(*ZpI*) + – min(*ZpI*)] = 0	3.1 – [max(*ZpI*) + – min(*ZpI*)] = 0	2.6 – [max(*ZpI*) + – min(*ZpI*)] = 0
	Yolk		
Diameter	32.2 – [max(*XżI*) + – min(*XżI*)]= 0 32.2 – [max(*YżI*) + – min(*YżI*)]= 0 32.2 – [max(*ZżI*) + – min(*ZżI*)]= 0	29.8 – [max(*XżII*) + – min(*XżII*)]= 0 29.8 – [max(*YżII*) + – min(*YżII*)]= 0 29.8 – [max(*ZżII*) + – min(*ZżII*)]= 0	29.1 – [max(*XżIII*) + – min(*XżIII*)]= 0 29.1 – [max(*YżIII*) + – min(*YżIII*)]= 0 29.1 – [max(*ZżIII*) + – min(*ZżIII*)]= 0

The analysis of the results included in table 1 shows that preciseness of chicken egss II and III is sufficient for practical purposes because a relative error in the indicated cross sections of the egg II is within -5.4 to 5.9% and for the model of III egg within 0 to 3.1.%. In case of the I model of egg a relative error of mapping, which is 10.9% occurred at the length being at the distance of 10 mm. Tables 3 and 4 show that the suggested method precisely maps the basic dimensions of the composing parts of chicken eggs (shell, yolk, air chamber).

Conclusions

1. Bézier patches may be used for modelling shapes of a shell, yolk and air chamber of a chicken egg.
2. The developed model *3D* of the chicken egg body which mapps the shell shape may serve for representing real eggs everywhere a great precision of mapping the shape is not required.
3. Mapped Bézier patches of the body of the composing parts of chicken eggs have identical basic dimensions of a shell (long axis, short axis I, short axis II, yolk surface (diameter) and air chamber (length, width, height) as corresponding chicken eggs.
4. The suggested method of modelling particular components may facilitate mapping the real shape of chicken eggs, e.g. in various states of their freshness may be used by designers for construing conveyors and separators.

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METODA MATEMATYCZNEGO MODELOWANIA KSZTAŁTU POWIERZCHNI SKORUPY, ŻÓŁTKA I KOMORY POWIETRZNEJ JAJA KURZEGO

Streszczenie. Przedstawiono metodę matematycznego modelowania kształtu powierzchni skorupy, żółtka i komory powietrznej jaja. Do modelowania kształtu wybrano jaja kurze o wymiarach: długość 60; 57,2; 56,4 mm, szerokość 47,1; 42,3; 41 mm i grubość 46; 41,1; 39,2 mm. Do odwzorowania kształtu powierzchni skorupy, żółtka i komory powietrznej jaja wykorzystano dwa płaty Béziera. Obliczenia i wizualizację zrealizowano w programie komputerowym Mathcad. Opracowany matematyczny model 3*D* kształtu jaja kurzego i jego podstawowych elementów można wykorzystać do modelowania i sterowania operacjami technologicznymi procesów produkcji i przetwarzania jaj.

Słowa kluczowe: jajo kurze, skorupa, żółtko, komora powietrzna, kształt, powierzchnia, płaty Béziera, model matematyczny.